ME-221
PROBLEM SET 8-9

Questions in red can only be answered after lecture 9. There will be no additional problem set next week.

Problem 1

The impulse response of a dynamical system is given by :

$$g(t) = 2\varepsilon(t)e^{-2t}$$

a) Determine the transfer function of the system.

b) Calculate and plot the unit step response of the system.

c) What is the time constant and steady-state (DC) gain of the system?

d) When does the output reach 95% of its steady-state value?

e) Calculate the response of the system for an input given by $u(t) = \varepsilon(t-1)e^{-(t-1)}$.

Problem 2

Consider the following dynamical system:

$$\ddot{y}(t) + k\dot{y}(t) + 4y(t) = u(t), \qquad y(0) = \dot{y}(0) = 0$$

a) Calculate the transfer function of the system. Determine the steady state (DC) gain, natural frequency, and damping ratio of the system.

b) Determine the form of the response (i.e. overdamped or underdamped) for $-5 < k < 5$.

Problem 3

The experimental measurements of the unit-step response of a dynamical system are shown in Table 1.

t [s]	0	...	1.8	2	2.2	2.4	2.6	2.8	3	3.2	3.4
y(t)	0	...	0	0	0.349	0.455	0.486	0.496	0.499	0.5	0.5

Table 1: Experimental Measurements

a) Plot the experimental values. Evaluate the graph along with given measurements to determine the order of the response and the corresponding transfer function.

b) Determine the impulse response in an analytical form.

Problem 4

Consider a system with the following transfer function:

$$G(s) = \frac{s-1}{(2s+1)(s+3)}$$

a) Calculate and plot the impulse response of the system.

b) Find the poles and zeros of the system. Is this system stable?

Problem 5

The unit step response of a second order dynamical system is shown in Figure 1.

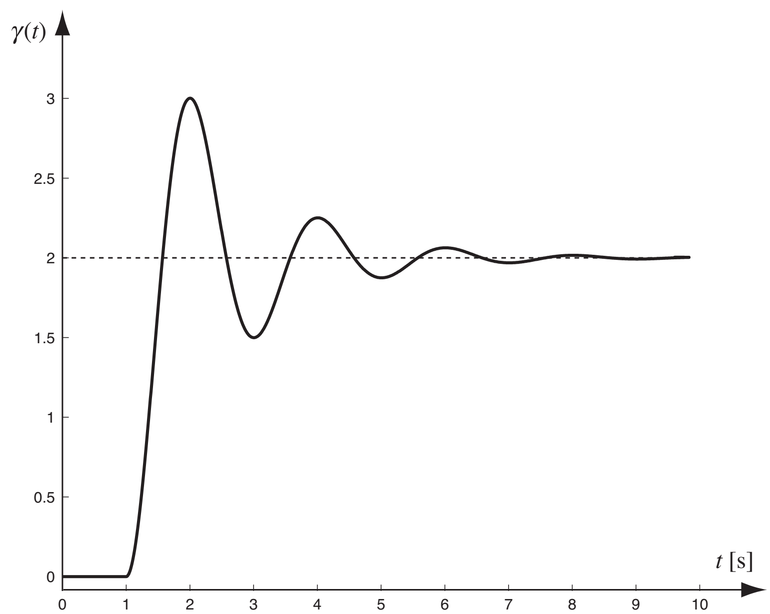

Figure 1: Transient response in the time domain

a) Report the values of the steady-state gain, damping ratio, natural frequency, and the poles of the system.

b) Determine the transfer function of the system given that the system has no zeros.

Problem 6

Consider the mechanical system shown in Figure 2. The system is described by the spring coefficient $k = 3500\ N/m$ and the viscous damping coefficient $f = 350\ Ns/m$, while the mass of the platform A-B is given by $m = 10\ kg$. A kid with mass $M = 90\ kg$ starts hanging on the platform at $t = 0$.

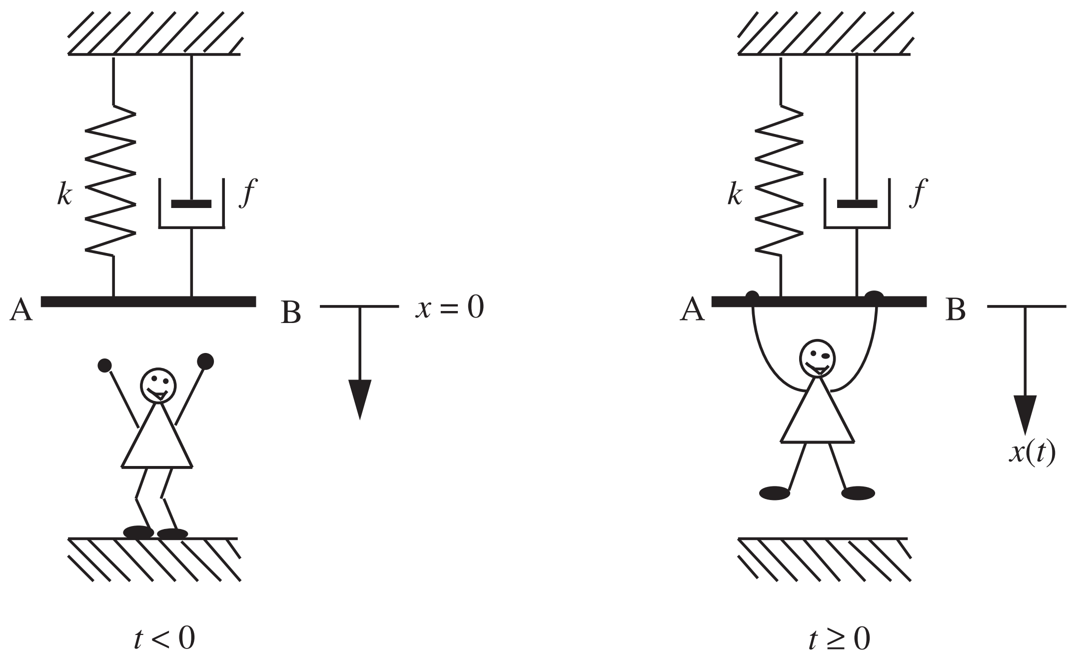

Figure 2: Mass Spring Damper System

a) Taking the weight of the kid as input and the position of the mass $x(t)$ as output, develop a mathematical model of the system.

b) Determine the order of the system. Calculate the poles and zeros.

c) Calculate the natural frequency and the damping ratio of the system.

d) Calculate the output for $t > 0$.